EX3

$C_1 = C_g$ $\quad g(0) = 1$

$C_2 = C_k$ $\quad k(0) = 2$

$C_3 = C_h$ $\quad h \geq 0$ et $\lim\limits_{-\infty} h = 0$

$C_4 = C_f$ $\quad f(x) < 0$

EX4

$$S_n = \sum_{k=0}^{n} \frac{1}{e^k}$$

Soit $U_n = \dfrac{1}{e^n} = e^{-n}$

(U_n) est une suite géométrique de raison $\dfrac{1}{e}$

$$S_n = \sum_{k=0}^{n} \frac{1}{e^k} = U_0 \left(\frac{1-(\frac{1}{e})^n}{1-\frac{1}{e}}\right)$$

$$= \frac{e}{e-1}\left[1-(\frac{1}{e})^{n+1}\right]$$

$\lim\limits_{n \to +\infty} (\dfrac{1}{e})^{n+1} = 0$ car $-1 < \dfrac{1}{e} < 1$

d'où $\lim\limits_{x \to +\infty} S_n = \dfrac{e}{e-1}$

EX5

* $\lim\limits_{x \to +\infty} (x^2 - e^x) =$

$\lim\limits_{x \to +\infty} x^2 (1 - \dfrac{e^x}{x^2}) = -\infty$

* $\lim\limits_{x \to +\infty} (e^{3x} - 2e^x) =$

$\lim\limits_{x \to +\infty} e^x (e^{2x} - 2) = +\infty$

* $\lim\limits_{x \to +\infty} (2e^x - \overbrace{\dfrac{1}{x}}^{\to 0}) = +\infty$

* $\lim\limits_{x \to -\infty} (2e^x - \dfrac{1}{x}) = 0$

* $\lim\limits_{x \to -\infty} xe^{-(1-x)} = \lim\limits_{x \to -\infty} \dfrac{xe^x}{e} = 0$

* $\lim\limits_{x \to -\infty} (x^2 - 2x + 3) e^x =$

$\lim\limits_{x \to -\infty} (\overbrace{x^2 e^x}^{0} - \overbrace{2x e^x}^{0} + \overbrace{3 e^x}^{0}) = 0$

* $\lim\limits_{x \to -\infty} (x^2 - x) e^x = \lim\limits_{x \to -\infty} (x^2 e^x - x e^x) = 0$

* $\lim\limits_{x \to 0^+} x\, e^{\frac{1}{x}} = \lim\limits_{t \to +\infty} \dfrac{e^t}{t} = +\infty$

(on pose $t = \dfrac{1}{x}$)

* $\lim\limits_{x \to +\infty} (x^3 - e^{2x}) = \lim\limits_{x \to +\infty} x^3 [1 - \dfrac{\overbrace{e^{2x}}^{\to +\infty}}{x^3}] = -\infty$

* $\lim\limits_{x \to +\infty} \dfrac{e^x}{\sqrt{x+1}} =$

$\lim\limits_{x \to +\infty} \dfrac{e^x}{\sqrt{x}\sqrt{1+\frac{1}{x}}} =$

$\lim\limits_{x \to +\infty} \left(\dfrac{e^x}{\sqrt{x}}\right) \dfrac{1}{\sqrt{1+\frac{1}{x}}} = +\infty$

$\lim\limits_{x \to +\infty} \dfrac{2x - e^x}{x^2 + x} = \lim\limits_{x \to +\infty} \dfrac{\frac{2}{x} - \frac{e^x}{x^2}}{1 + \frac{1}{x}} = -\infty$

On pose $X = -\dfrac{1}{x}$

* $\lim\limits_{x \to 0^+} \dfrac{1}{x} e^{-\frac{1}{x}} = \lim\limits_{X \to -\infty} -X e^X = 0$

* $\lim\limits_{x \to +\infty} \dfrac{e^x - 1}{x} = \lim\limits_{x \to +\infty} \dfrac{e^x}{x} - \dfrac{1}{x} = +\infty$

Qarray.tn -Aqra Blech- Qarray.tn